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Reflectionless spin waves in Heisenberg chains with spin impurity segments

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Abstract. We study, via a scattering transfer-matrix approach, the spin waves of a Heisenberg chain with randomly inserted impurity segments having spin and coupling strength different from the host ones. Numerical results show the existence of completely unscattered reflectionless modes, whose number is sensitive to the impurity parameters. If both the spin and the coupling strength of the impurities are different from those of the host we find a novel type of resonant propagating wave, for which the reflection by a single segment is minimum but not vanishing. The relevance of the present results to other systems and to possible applications are discussed.

1. Introduction

Recently, there has been intensive interest in wave propagation in systems with disorder in one direction. The purpose is to search for localization of the optical or acoustic waves in a real system [1, 2] and to explore possible applications of the random structures in new devices. In many cases the wave propagation can be characterized by a one-dimensional (1D) Hamiltonian which incorporates the disorder. From scaling theory it has been established that in 1D disordered systems the diffusion coefficient vanishes and the localization length is finite even for an infinitesimal amount of disorder [3]. However, in recent theoretical investigations it was shown that there are several types of 1D disordered systems in which a small number of extended states exists [4–10]. Generally speaking, for the existence of extended electronic states in 1D disordered systems, it is essential to introduce correlations in the disorder, i.e. the random parameters on every site should depend on the corresponding values of their neighbours within a correlation length [10].

It is well known [11] that the magnon equations of motion in ferromagnetic spin chains with disorder in the exchange strengths can be mapped exactly onto an electronic chain with a particular form of off-diagonal disorder where the random bonds also appear in pairs. Therefore, a magnon system should display similar properties to a correlated electronic system. However, this exact mapping of the magnon equations to a correlated electronic system fails in a Heisenberg chain if there are impurities with different spins from the host. This produces special forms of diagonal and off-diagonal disorder in the magnon system. In the rest of this paper we investigate a system with both spin and exchange-strength randomnesses and compare the corresponding effects on spin wave propagation.

In this paper we study the spin waves of a Heisenberg chain with randomly inserted impurity segments in which the spins and the coupling strengths may be optionally different

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from those of the other parts of the chain. We show that there exist reflectionless spin wave modes, similar to those found in 1D correlated electronic systems [12–15]. It turns out that their number is sensitive to the spin value and the coupling strength of the impurities. Moreover, we show that if both the spin and the coupling strength of the impurities are different from those of the host chain, another type of resonant propagating wave can exist for which the reflection by a single segment is minimum but not vanishing. This case corresponds to the so-called 'incomplete reflectionless modes' and an incomplete resonant transmission is produced in a random system with a finite number of inserted segments.

2. Model and formalism

The system we study is a 1D chain constructed by randomly inserting a number of identical segments, each of which contains l sites of spin s_1, into an originally pure chain, in which the site spin is s_0. So the chain is divided by spacing segments with random lengths. We assume that the nearest-neighbour exchange strength is J_1 inside the inserted segment and at the connection of two segments, and is J_0 inside the spacing segment. The inserted segments and the spacing segments are alternately connected. We use the index j to number them and assign an odd j to the former and an even j to the latter. If the sites within the jth segment have the same spin, denoted by S_j, the Hamiltonian can be expressed as

$$H = \sum_n J_n S_n \cdot S_{n+1} \tag{1}$$

where

$$S_n \equiv S_j = \begin{cases} s_0 & m_j + 1 \leqslant n \leqslant m_j + l_j \qquad j \text{ even} \\ s_1 & m_j + 1 \leqslant n \leqslant m_j + l_j \qquad j \text{ odd} \end{cases}$$

and

$$J_n = \begin{cases} J_0 & S_n = S_{n+1} = s_0 \\ J_1 & \text{otherwise} \end{cases}$$

with $m_j + 1$ and l_j being the first site and the length of the jth segment, respectively. In the studied structures the lengths of the inserted segments are the same, i.e.

$$l_j = l \qquad j = 2i + 1$$

with i an integer, and the randomness lies in the distribution of the lengths of the spacing segments, which we choose to be distributed according to the uniform function

$$P(l_j) = \begin{cases} 1/a & |l_j - a_0| \leqslant a/2 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $j = 2i$, a_0 is the average length and a is the spreading width of the random lengths. The strength of the randomness is parametrized by a.

The general one-magnon state can be written as $\sum_n c_n |n\rangle$, with $|n\rangle$ being the state with spin on the nth site reduced by one unit. The c_n satisfy the following difference equation:

$$\epsilon_n c_n - V_{n-1} c_{n-1} - V_n c_{n+1} = E c_n \tag{3}$$

where

$$\epsilon_n = J_{n-1}S_{n-1} + J_n S_{n+1} \qquad V_n = J_n\sqrt{S_n S_{n+1}}$$

and E is the energy of the magnon. Within the jth segment, there are l_j coefficients c_n ($n = m_j + 1, m_j + 2, ..., m_j + l_j$), and $l_j - 2$ equations for them which have the same parameter values for the spin and exchange strength. As a result, for a segment with $l_j \geqslant 2$, the magnon state is characterized by two independent coefficients, denoted by A_j and B_j, and the c_n are related to them by

$$c_n = A_j \cos[k_j(n - m_j - 1)] + B_j \sin[k_j(n - m_j - 1)] \qquad m_j + 1 \leqslant n \leqslant m_j + l_j \tag{4}$$

where $\cos(k_j) = 1 - (E/2S_j J_j)$, with J_j being J_0 for even j and J_1 for odd j. At the connection between the jth segment and the $(j + 1)$th segment, there are two unused equations, which lead to the following relation:

$$\hat{M}_{j+1}\begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix} = \hat{N}_j \hat{\Lambda}_j \begin{pmatrix} A_j \\ B_j \end{pmatrix} \tag{5}$$

where $\hat{M}_j$, $\hat{N}_j$ and $\hat{\Lambda}_j$ are the 2×2 matrices

$$\hat{M}_j = \begin{pmatrix} J_j\sqrt{s_0 s_1} & 0 \\ E - J_j(s_0 + s_1) + J_j S_j \cos(k_j) & J_j S_j \sin(k_j) \end{pmatrix}$$

$$\hat{N}_j = \begin{pmatrix} -E + J_0 s_0 + J_1 s_1 - J_j S_j \cos(k_j) & J_j S_j \sin(k_j) \\ -J_{j+1}\sqrt{s_0 s_1} & 0 \end{pmatrix}$$

$$\hat{\Lambda}_j = \begin{pmatrix} \cos[k_j(l_j - 1)] & \sin[k_j(l_j - 1)] \\ -\sin[k_j(l_j - 1)] & \cos[k_j(l_j - 1)] \end{pmatrix}.$$

If $l_j = 1$, there is only one coefficient in the segment. We can introduce a virtual coefficient, say B_j, and an additional equation for it to keep (5) unchanged.

3. Reflectionless spin waves in a random chain

If there is only one inserted segment, the coefficients of the spin wave in the other part of the chain can be expressed as

$$c_n = \begin{cases} \exp[\mathrm{i}k_0(n - m_1)] + r\exp[-\mathrm{i}k_0(n - m_1)] & n \leqslant m_1 \\ t\exp[\mathrm{i}k_0(n - m_1 - l_1 - 1)] & n \geqslant m_1 + l_1 + 1 \end{cases} \tag{6}$$

where r is the reflection coefficient and t is the transmission coefficient.

The coefficients defined in the last section, A_0, B_0, A_2 and B_2, are related to r and t by

$$A_0 = r + 1 \qquad B_0 = \mathrm{i}(1 - r) \qquad A_2 = t \qquad B_2 = \mathrm{i}t. \tag{7}$$

From (5), one has

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = \hat{T}\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} \tag{8}$$

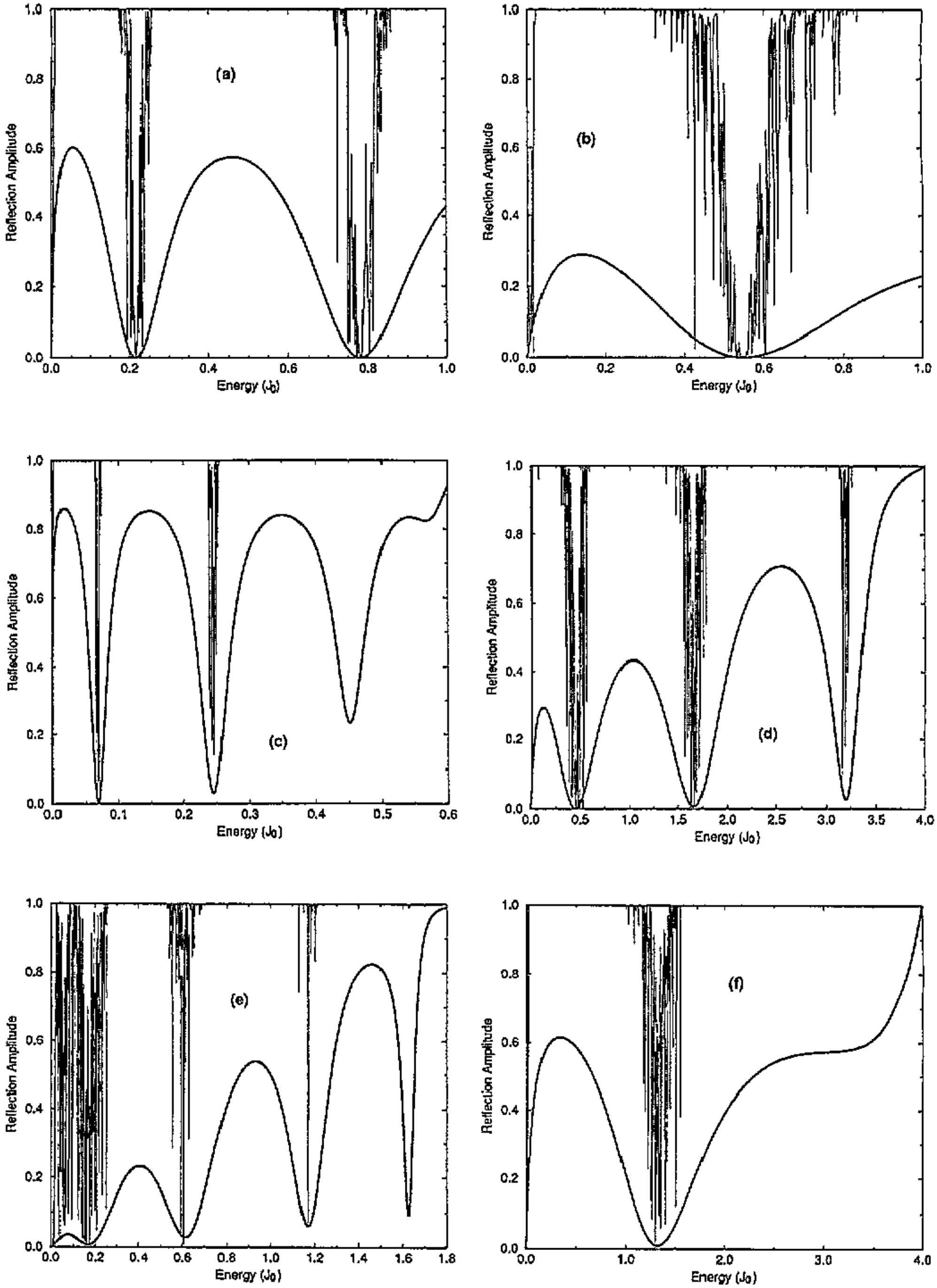

Figure 1. Calculated reflection amplitude $|r|^2$ as a function of spin wave energy for various values of J_1 and s_1. Thick curve: system with only a single impurity segment; thin curve: system with 80 randomly inserted segments, with $s_0 = J_0 = 1$. The length of the inserted segments is $l = 5$ for figures (*a*)–(*g*). The parameters in the probability (2) are $a_0 = 11$, $a = 4$. (*a*) $J_1 = 1$, $s_1 = 0.5$; (*b*) $J_1 = 1$, $s_1 = 1.5$; (*c*) $J_1 = 0.3$, $s_1 = 0.5$; (*d*) $J_1 = 2.5$, $s_1 = 0.5$; (*e*) $J_1 = 0.3$, $s_1 = 1.5$; (*f*) $J_1 = 2.5$, $s_1 = 1.5$; (*g*) $J_1 = 2.5$, $s_1 = 1$; (*h*) $J_1 = 0.3$, $s_1 = 1$; (*i*) $J_1 = 2.5$, $s_1 = 1$, $l = 10$.

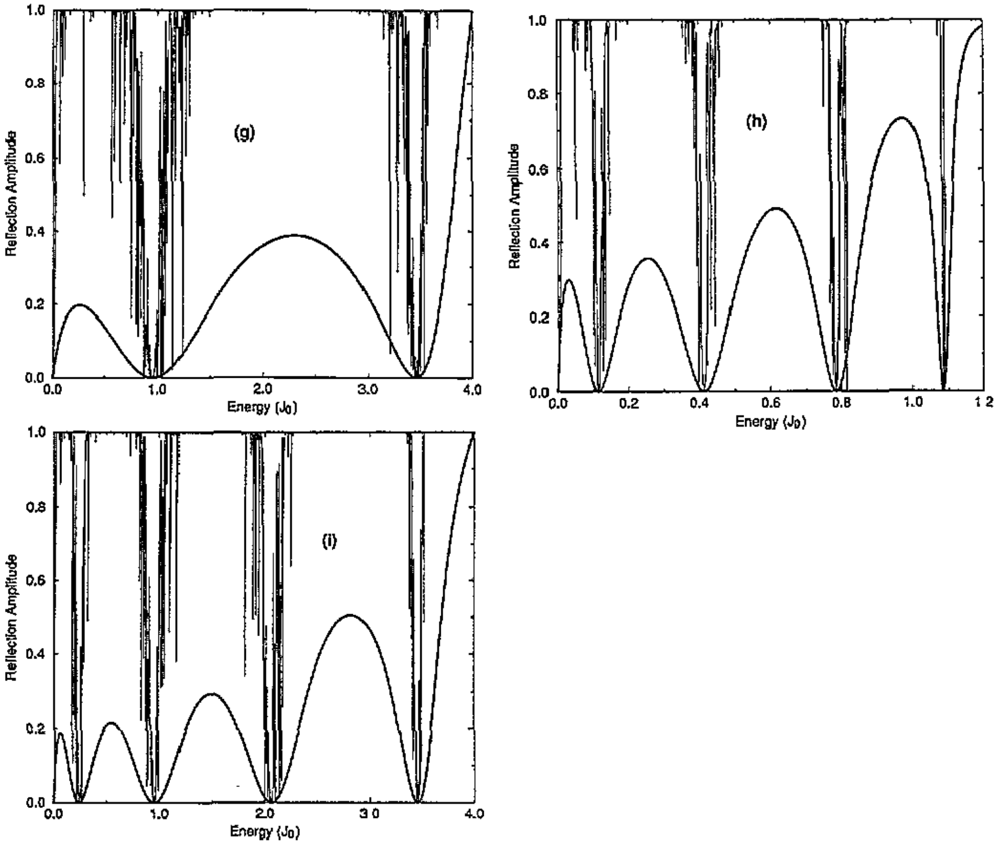

Figure 1. Continued.

where

$$\hat{T} = \hat{M}_0^{-1}\hat{N}_1\hat{\Lambda}_1\hat{M}_1^{-1}\hat{N}_0. \tag{9}$$

By combining (7) and (8) one yields

$$r = \frac{T_{22} - T_{11} - \mathrm{i}T_{21} - \mathrm{i}T_{12}}{T_{11} + T_{22} + \mathrm{i}T_{21} - \mathrm{i}T_{12}} \tag{10}$$

where $T_{pp'}$ $(p, p' = 1, 2)$ are elements of the 2×2 matrix $\hat{T}$.

The reflection amplitude $|r|^2$ can be calculated from (9) and is a function of the energy of magnons. At certain special modes $r = 0$ is satisfied and the spin waves are not reflected by a single inserted segment. When many impurity segments of the same length are inserted at random the special modes remain reflectionless despite their random arrangement [10, 12]. In figure 1 we use the thick curves to illustrate the reflection amplitudes of spin waves by a single segment and the thin lines for many segments at random, as a function of the energy and for different choices of s_1, J_1 and l, with s_0 and J_0 set to unity. It can be seen that the propagation of spin waves is very sensitive to the differences $s_1 - s_0$ and $J_1 - J_0$. For a fixed length of the inserted segment, the reflectionless modes become denser and the average reflection amplitudes are increased when these differences are reduced. We also observe that the long-wavelength $E = 0$ mode always remains unscattered by the impurities.

The differences in the exchange strength and spin magnitude between the impurities and the host have almost the same effect on the number of the reflectionless modes, because both of them produce diagonal and off-diagonal randomness in the system. The observed non-zero minima in the reflection amplitude against energy become exactly zero, corresponding to completely reflectionless modes, when one of the $s_1 - s_0$, $J_1 - J_0$ becomes zero. On the contrary, if both of them are non-zero, the amplitudes at some minima are not completely vanishing, corresponding to another type of resonant mode. Moreover, if the length of the inserted impurity segment is increased, the number of the reflectionless modes and the resonant modes also increases [10, 12].

If there are more than one randomly inserted impurity segments, the total transfer matrix for the waves is

$$\hat{T}_{\text{total}} = \prod_i (\hat{T}\hat{\Lambda}_{2i}) \tag{11}$$

and the reflection coefficient is calculated from (9) replacing $\hat{T}$ by $\hat{T}_{\text{total}}$. In figure 1 we use the thin curves to illustrate the reflection amplitudes from many impurity segments for different values of s_1 and J_1. It can be seen that for a finite number of the inserted segments the minima in the curves for a single segment (thick curves) usually well reproduce the resonant transmission of the spin waves. By increasing the number and randomness of the inserted impurities the reflectionless modes are not influenced while the reflection amplitudes of the other modes drastically increase. If the total number of the inserted segments M is large enough, only the reflectionless modes can propagate through the system without significant damping. If M is small, the incomplete resonant modes can also produce resonant transmission but with transmission coefficient less than one. The variation of the reflection amplitude with the number of inserted segments and the randomness strength is illustrated in figure 2. We would also observe that the number of the reflectionless modes is sensitive to the differences in the spin and exchange strength between the impurities and the host. Generally speaking, there exist more reflectionless modes in the case of smaller $|J_1 - J_0|$ and small $|s_1 - s_0|$. If both of them vanish, the randomness disappears and all the modes propagate without reflection.

We also numerically investigate the energy dependence of the reflection amplitude in the vicinity of a reflectionless mode. We define the exponent for this dependence as

$$\alpha(E) = \ln(|r|^2)/\ln(|E - E_c|) \tag{12}$$

where E_c is the energy of the reflectionless mode. In figure 3 we plot the variation of the exponent α with energy for different choices of the parameters. We can see that for a completely reflectionless mode with $E_c \neq 0$, the exponent is exactly two at the resonant energy, independent of the randomness, the size of the system and the other parameters of the model. This behaviour may be related to the one-magnon approximation of the theory. On the other hand, the exponent for the special mode at $E = 0$ is always one, giving a linear energy dependence for the long-wavelength mode.

The resonant transmission found in the present disordered chain relies crucially on the assumption of identical inserted segments. We have also used the same approach to calculate the reflection amplitude as a function of energy for a system with the length segments randomly distributed. The result shows that almost all the reflectionless modes are now eliminated by the randomness in the segment lengths, except for the $E = 0$ mode which remains unscattered.

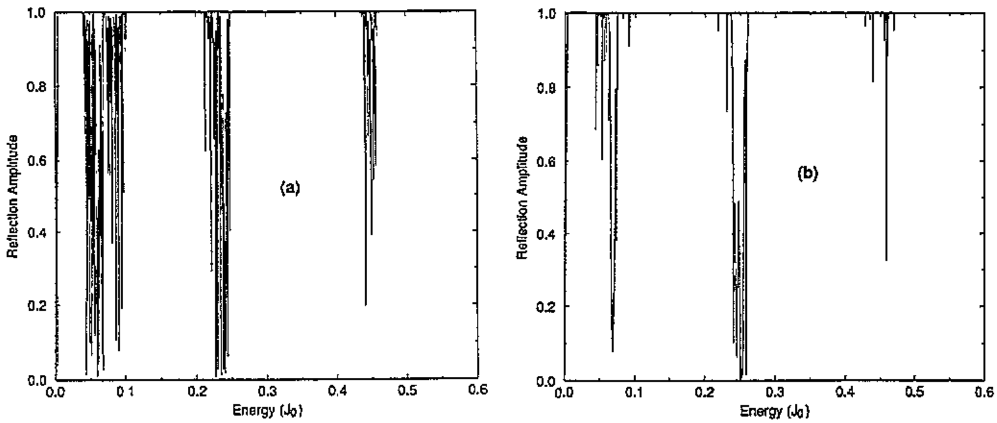

Figure 2. The reflection amplitude as a function of energy for a different number of inserted segments and different amounts of randomness. J_1 and s_1 are the same as those in figure 1(*c*). $l = 5$. (*a*) $a_0 = 11$, $a = 1$, $M = 80$. (*b*) $a_0 = 11$, $a = 4$, $M = 20$.

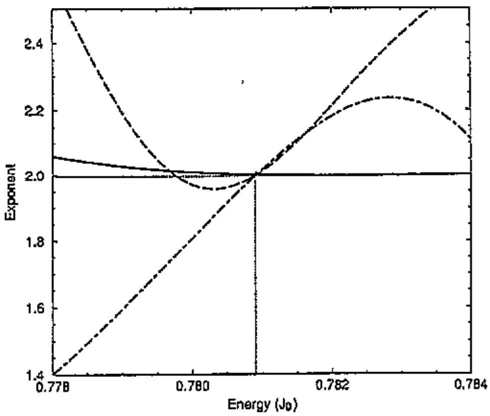

Figure 3. The exponent defined in (11) as a function of energy for the parameters of figure 1(*a*). The full curve is for the system with $M = 4$, the broken curve is for $M = 15$, and the chain curve is for $M = 30$.

4. Discussion

We studied a system which is constructed by randomly inserting impurity segments with both different spin and coupling strength into a host Heisenberg chain. As all the impurity segments are the same, a spin wave mode will not be reflected by the whole system if it remains unscattered by a single segment. At energies other than the special modes the spin waves are strongly reflected by the impurities. Therefore, a sharp resonance appears only at the special energies. This kind of resonance may be used to produce high-quality spin wave filters. If the spin and coupling strength of the impurities are both different from those of the host, we obtain another type of resonant mode, the so-called incomplete reflectionless mode. In the present model the transmissive parts of all but the complete reflectionless modes are expected to decay exponentially when the number of the inserted impurity segments is increased. However, for a sample with a finite number of impurity segments, the transmission coefficient of the incomplete reflectionless modes are less than 1 but may be finite, much larger than that of the other modes. This is due to the fact that the decay of these modes is much slower than that of the rest of the modes.

In this paper the exchange coupling at the interfaces is assumed to be the same as that within the segments. We have also investigated the more general situation with the exchange coupling J_3 at the interface between s_0 and s_1 being different from both the J_0 and J_1. By introducing this different J_3, the reflectionless modes (complete and incomplete) at special non-zero energies still exist, although their positions are shifted to different values compared to the case with $J_3 = J_0$. In the case of different J_3, these modes are also completely unscattered when $s_0 = s_1$ or $J_0 = J_1$. Thus, the existence of the reflectionless modes is a generic feature of the systems considered.

The present results are also relevant for other closely related classical wave systems, such as the phonon or proton, which can be simply introduced by replacing the magnon with the proton or phonon operators. An obvious extension of the present study is to introduce non-linearity in the model. In such a case it is expected that the special reflectionless modes may play an important role for soliton propagation in the presence of impurities [16, 17].

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